PROBLEM 3.103

Determine the single equivalent force and the distance from Point A to its line of action for the beam and loading of (a) Prob. 3.101a, (b) Prob. 3.101b, (c) Prob. 3.102.

SOLUTION

For equivalent single force at distance \( d \) from \( A \):

(a) We have \( \Sigma F_y: \quad -300 \, \text{N} - 200 \, \text{N} = R \)

\[ \text{or } R = 500 \, \text{N} \]

and \( \Sigma M_c: \quad -400 \, \text{N} \cdot \text{m} + (300 \, \text{N})(d) \)

\[ - (200 \, \text{N})(3 - d) = 0 \]

\[ \text{or } d = 2.00 \, \text{m} \]

(b) We have \( \Sigma F_y: \quad 200 \, \text{N} + 300 \, \text{N} = R \)

\[ \text{or } R = 500 \, \text{N} \]

and \( \Sigma M_c: \quad -400 \, \text{N} \cdot \text{m} - (200 \, \text{N})(d) \)

\[ + (300 \, \text{N})(3 - d) = 0 \]

\[ \text{or } d = 1.000 \, \text{m} \]

(c) We have \( \Sigma F_y: \quad -200 \, \text{N} - 300 \, \text{N} = R \)

\[ \text{or } R = 500 \, \text{N} \]

and \( \Sigma M_c: \quad 500 \, \text{N} \cdot \text{m} + 200 \, \text{N} \cdot \text{m} \)

\[ + (200 \, \text{N})(d) - (300 \, \text{N})(3 - d) = 0 \]

\[ \text{or } d = 0.400 \, \text{m} \]
PROBLEM 3.106

Three stage lights are mounted on a pipe as shown. The lights at A and B each weigh 4.1 lb, while the one at C weighs 3.5 lb. (a) If \( d = 25 \) in., determine the distance from D to the line of action of the resultant of the weights of the three lights. (b) Determine the value of \( d \) so that the resultant of the weights passes through the midpoint of the pipe.

**SOLUTION**

For equivalence,

\[
\Sigma F_y : -4.1 - 4.1 - 3.5 = -R \quad \text{or} \quad R = 11.7 \text{ lb}
\]

\[
\Sigma F_D: - (10 \text{ in})(4.1 \text{ lb}) - (44 \text{ in})(4.1 \text{ lb})
\]

\[
-[(4.4 + d) \text{ in}](3.5 \text{ lb}) = -(L \text{ in})(11.7 \text{ lb})
\]

or

\[375.4 + 3.5d = 11.7L \quad (d, L \text{ in } \text{in})\]

(a)

\[d = 25 \text{ in.}\]

We have

\[375.4 + 3.5(25) = 11.7L \quad \text{or} \quad L = 39.6 \text{ in.}\]

The resultant passes through a point 39.6 in. to the right of D.

(b)

\[L = 42 \text{ in.}\]

We have

\[375.4 + 3.5d = 11.7(42) \quad \text{or} \quad d = 33.1 \text{ in.}\]
PROBLEM 3.118

As follower \( AB \) rolls along the surface of member \( C \), it exerts a constant force \( F \) perpendicular to the surface. (a) Replace \( F \) with an equivalent force-couple system at Point \( D \) obtained by drawing the perpendicular from the point of contact to the \( x \)-axis. (b) For \( a = 1 \text{ m} \) and \( b = 2 \text{ m} \), determine the value of \( x \) for which the moment of the equivalent force-couple system at \( D \) is maximum.

SOLUTION

(a) The slope of any tangent to the surface of member \( C \) is

\[
\frac{dy}{dx} = \frac{d}{dx} \left[ b \left(1 - \frac{x^2}{a^2}\right)\right] = \frac{-2b}{a^2} x
\]

Since the force \( F \) is perpendicular to the surface,

\[
\tan \alpha = \left( \frac{dy}{dx} \right)^{-1} = \frac{a^2}{2b} \left( \frac{1}{x} \right)
\]

For equivalence,

\[
\Sigma F: \quad F = R
\]

\[
\Sigma M_D: \quad (F \cos \alpha)(y_C) = M_D
\]

where

\[
\cos \alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}
\]

\[ y_C = b \left(1 - \frac{x^2}{a^2}\right) \]

\[ 2Fb^2 \left( \frac{x - \frac{x^3}{a^2}}{a} \right) \]

\[ M_D = \frac{2Fb^2 \left( x - \frac{x^3}{a^2} \right)}{\sqrt{a^4 + 4b^2x^2}} \]

Therefore, the equivalent force-couple system at \( D \) is

\[ R = F \tan^{-1} \left( \frac{a^2}{2bx} \right) \]

\[ 2Fb^2 \left( x - \frac{x^3}{a^2} \right) \]

\[ M = \frac{2Fb^2 \left( x - \frac{x^3}{a^2} \right)}{\sqrt{a^4 + 4b^2x^2}} \]
PROBLEM 3.123

A blade held in a brace is used to tighten a screw at A. (a) Determine the forces exerted at B and C, knowing that these forces are equivalent to a force-couple system at A consisting of \( \mathbf{R} = -(30 \text{ N}) \mathbf{i} + R_y \mathbf{j} + R_z \mathbf{k} \) and \( \mathbf{M}_A = -(12 \text{ N} \cdot \text{ m}) \mathbf{i} \). (b) Find the corresponding values of \( R_y \) and \( R_z \). (c) What is the orientation of the slot in the head of the screw for which the blade is least likely to slip when the brace is in the position shown?

SOLUTION

(a) Equivalence requires
\[ \sum \mathbf{F} = \mathbf{R} = \mathbf{B} + \mathbf{C} \]

or
\[ -(30 \text{ N}) \mathbf{i} + R_y \mathbf{j} + R_z \mathbf{k} = -B \mathbf{k} + (-C_x \mathbf{i} + C_y \mathbf{j} + C_z \mathbf{k}) \]

Equating the i coefficients:
\[ 1: \quad -30 \text{ N} = -C_x \quad \text{or} \quad C_x = 30 \text{ N} \]

Also,
\[ \sum \mathbf{M}_A = \mathbf{M}_A^B = r_{BA} \times \mathbf{B} + r_{CA} \times \mathbf{C} \]

or
\[ -(12 \text{ N} \cdot \text{ m}) \mathbf{i} = [(0.2 \text{ m}) \mathbf{i} - (0.15 \text{ m}) \mathbf{j}] \times (-B) \mathbf{k} \]
\[ + (0.4 \text{ m}) \mathbf{i} \times [-30 \text{ N} \mathbf{i} + C_y \mathbf{j} + C_z \mathbf{k}] \]

Equating coefficients:
\[ i: \quad -12 \text{ N} \cdot \text{ m} = -(0.15 \text{ m})B \quad \text{or} \quad B = 80 \text{ N} \]

\[ k: \quad 0 = (0.4 \text{ m})C_z \quad \text{or} \quad C_z = 0 \]

\[ j: \quad 0 = (0.2 \text{ m})(80 \text{ N}) - (0.4 \text{ m})C_z \quad \text{or} \quad C_z = 40 \text{ N} \]

\[ B = -(80.0 \text{ N}) \mathbf{k} \quad C = -(30.0 \text{ N}) \mathbf{i} + (40.0 \text{ N}) \mathbf{k} \]

(b) Now we have for the equivalence of forces
\[ -(30 \text{ N}) \mathbf{i} + R_y \mathbf{j} + R_z \mathbf{k} = -(80 \text{ N}) \mathbf{k} + [-30 \text{ N} \mathbf{i} + (40 \text{ N}) \mathbf{k}] \]

Equating coefficients:
\[ j: \quad R_y = 0 \quad R_y = 0 \]

\[ k: \quad R_z = -80 + 40 \quad \text{or} \quad R_z = -40.0 \text{ N} \]

(c) First note that \( \mathbf{R} = -(30 \text{ N}) \mathbf{i} - (40 \text{ N}) \mathbf{k} \). Thus, the screw is best able to resist the lateral force \( R_z \) when the slot in the head of the screw is vertical.
PROBLEM 3.145*

Show that a wrench can be replaced with two perpendicular forces, one of which is applied at a given point.

SOLUTION

![Diagram showing wrench forces](image)

First, observe that it is always possible to construct a line perpendicular to a given line so that the constructed line also passes through a given point. Thus, it is possible to align one of the coordinate axes of a rectangular coordinate system with the axis of the wrench while one of the other axes passes through the given point.

See Figures a and b.

We have \( \mathbf{R} = R \mathbf{j} \) and \( \mathbf{M} = M \mathbf{j} \) and are known.

The unknown forces \( \mathbf{A} \) and \( \mathbf{B} \) can be expressed as

\[
\mathbf{A} = A_x \mathbf{i} + A_y \mathbf{j} + A_z \mathbf{k}
\]

and

\[
\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}
\]

The distance \( a \) is known. It is assumed that force \( \mathbf{B} \) intersects the \( xz \)-plane at \( (x, 0, z) \). Then for equivalence,

\[
\begin{align*}
\Sigma F_x & : 0 = A_x + B_x \\
\Sigma F_y & : R = A_y + B_y \\
\Sigma F_z & : 0 = A_z + B_z \\
\Sigma M_x & : 0 = -zB_y \\
\Sigma M_y & : M = -aA_z - xB_z + zB_x \\
\Sigma M_z & : 0 = aA_y + xB_y
\end{align*}
\]

Since \( \mathbf{A} \) and \( \mathbf{B} \) are made perpendicular,

\[
\mathbf{A} \cdot \mathbf{B} = 0 \quad \text{or} \quad A_x B_x + A_y B_y + A_z B_z = 0 \quad (7)
\]

There are eight unknowns: \( A_x, A_y, A_z, B_x, B_y, B_z, x, z \)

But only seven independent equations. Therefore, \textit{there exists an infinite number of solutions.}

Next, consider Equation (4):

\[
0 = -zB_y
\]

If \( B_y = 0 \), Equation (7) becomes

\[
A_x B_x + A_z B_z = 0
\]

Using Equations (1) and (3), this equation becomes

\[
A_x^2 + A_z^2 = 0
\]
PROBLEM 3.145* (Continued)

Since the components of \( \mathbf{A} \) must be real, a nontrivial solution is not possible. Thus, it is required that \( B_y \neq 0 \), so that from Equation (4), \( z = 0 \).

To obtain one possible solution, arbitrarily let \( A_z = 0 \).

(Note: Setting \( A_y, A_z, \) or \( B_z \) equal to zero results in unacceptable solutions.)

The defining equations then become

\[
\begin{align*}
0 &= B_z \quad \text{(1)' } \\
R &= A_y + B_y \\
0 &= A_z + B_z \\
M &= -aA_z - xB_z \\
0 &= aA_y + xB_y \\
A_yB_y + A_zB_z &= 0 \\
\end{align*}
\]

Then Equation (2) can be written

\[
A_y = R - B_y
\]

Equation (3) can be written

\[
B_z = -A_z
\]

Equation (6) can be written

\[
x = -\frac{aA_y}{B_y}
\]

Substituting into Equation (5'),

\[
M = -aA_z - \left( -a \frac{R - B_y}{B_y} \right)(-A_z)
\]

or

\[
A_z = -\frac{M}{aR}B_y
\]

Substituting into Equation (7'),

\[
(R - B_y)B_y + \left( -\frac{M}{aR}B_y \right) \left( \frac{M}{aR}B_y \right) = 0
\]

or

\[
B_y = \frac{a^2 R^3}{a^2 R^2 + M^2}
\]

Then from Equations (2), (8), and (3),

\[
\begin{align*}
A_y &= R - \frac{a^2 R^2}{a^2 R^2 + M^2} = \frac{RM^2}{a^2 R^2 + M^2} \\
A_z &= \frac{M}{aR} \left( -\frac{a^2 R^3}{a^2 R^2 + M^2} \right) = -\frac{a^2 R^3 M}{a^2 R^2 + M^2} \\
B_z &= \frac{a^2 R^3 M}{a^2 R^2 + M^2}
\end{align*}
\]
In summary,

\[ A = \frac{RM}{a^2 R^2 + M^2} (M_j - aRk) \]

\[ B = \frac{aR^2}{a^2 R^2 + M^2} (aR_j + Mk) \]

Which shows that it is possible to replace a wrench with two perpendicular forces, one of which is applied at a given point.

Lastly, if \( R > 0 \) and \( M > 0 \), it follows from the equations found for \( A \) and \( B \) that \( A_y > 0 \) and \( B_y > 0 \).

From Equation (6), \( x < 0 \) (assuming \( a > 0 \)). Then, as a consequence of letting \( A_x = 0 \), force \( A \) lies in a plane parallel to the \( yz \)-plane and to the right of the origin, while force \( B \) lies in a plane parallel to the \( yz \)-plane but to the left of the origin, as shown in the figure below.